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HARMONIC FUNCTIONS ON REGIONS WITH REENTRANT CORNERS. PART I.(U)

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HARMONIC FUNCTIONS ON REGIONS WITH REENTRANT CORNERS, PART I

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J. Barkley Rosser

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ABSTRACT

It has been known for quite a while that if a function $u(x,y)$ is harmonic in a region with reentrant corners, there are almost certainly infinite discontinuities of the first derivative of u in the neighborhood of the reentrant corner (or corners). Simple examples are for an L-shaped region or T-shaped region. Some instances of these have been treated by conformally mapping the region into the interior of a rectangle. Attempts to solve the problem as first posed by a finite difference scheme or a finite element scheme will usually give poor approximations near any reentrant corner because the finite differences or finite elements have large truncation errors when a first derivative is infinite. When conformal mapping is tried, the conformal maps are usually only approximate, and similar errors arise, for more or less similar reasons.

In view of recent work giving convergent expansions for u in the neighborhood of reentrant corners (see "Calculation of Potential in a Sector, Part I," by J. Barkley Rosser, MRC TSR #1535) one can now give accurate solutions for such problems. Some experiments with such regions are reported.

AMS (MOS) Subject Classification: 65N99

Key Words: Partial differential equations, Boundary value problems, Numerical solution

Work Unit Number 7 (Numerical Analysis)

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SIGNIFICANCE AND EXPLANATION

There are numerous situations in which the behavior is given by a function which is harmonic in a region; temperature in the region, fluid flow through the region, diffusion through a permeable medium in the region, to name a few. If there are no obstacles in the region, there are numerous techniques which give the desired harmonic function efficiently. However, there are getting to be more and more cases in which the medium has obstacles in it, commonly obstacles with corners. Typically a corner of an obstacle will be a reentrant corner as far as the medium is concerned. That is, one must traverse more than 180° THROUGH THE MEDIUM to get from one edge of the angle to the other. It was discovered more than twenty years ago that, at such a reentrant corner, the harmonic function that one is seeking usually has infinite derivatives. In such case, the familiar techniques for finding the harmonic function give very poor approximations near the corner. The present paper gives a new technique which will give high accuracy approximations to the desired harmonic function without undue labor. Part I, which is presented here, gives the theoretical background of the technique. Part II, which will follow, will give numerical examples, to show how the technique works in practical situations.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

HARMONIC FUNCTIONS ON REGIONS WITH REENTRANT CORNERS, PART I

J. Barkley Rosser

1. Background. A function $u(x,y)$ is said to be harmonic if

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 . \tag{1.1}$$

We approximate (1.1) by the familiar difference formula

$$u(x + h,y) + u(x - h,y) + u(x,y + h) + u(x,y - h) - 4u(x,y) \cong 0 . \tag{1.2}$$

The error involves fourth derivatives of u. We suppose that these exist and are reasonably well behaved. This is usually the case, so for a long time it has been customary to seek an approximate solution for (1.1) by solving the set of linear equations resulting from using various values of (x,y) in (1.2). If h is small, so that the error in (1.2) is small, the solution of (1.2) gives good approximations to the values of u at a set of grid points.

Unfortunately, if h is small, then one has a very large number of linear equations to solve, and the labor of computation is very great. Until the advent of the computer, one compromised by using a fairly large value of h, to curtail the calculation, but one had to be content with not a very good approximation.

One effort to use computers to improve this situation is embodied in item [1] of the Bibliography, by Kantorovich, Krylov, and Chernin. If one has values for u prescribed on the boundary of a rectangle, the tables in [1] allow one to get fairly quickly the solution of (1.2) inside the rectangle. (This can now be done more quickly by means of the Fast Fourier Transform, so that [1] is now obsolete.)

To show the effectiveness of their tables, the authors of [1] undertook to find the solution of (1.2) inside an L-shaped region (see Fig. 1). They prescribed values for u around the boundary, and used the Schwarz alternating procedure. Specifically, they first guessed values along CF. Using these with the boundary conditions, the tables gave the solution of (1.2) inside the rectangle ABFG. In particular, they gave values along HC. Using these with the boundary conditions, the tables gave the solution of (1.2) inside the rectangle HDEG. This led to a better guess for the values along CF. Using the better guess, the process was repeated. After a modest number of repetitions, the procedure converged to give about six decimal accuracy.

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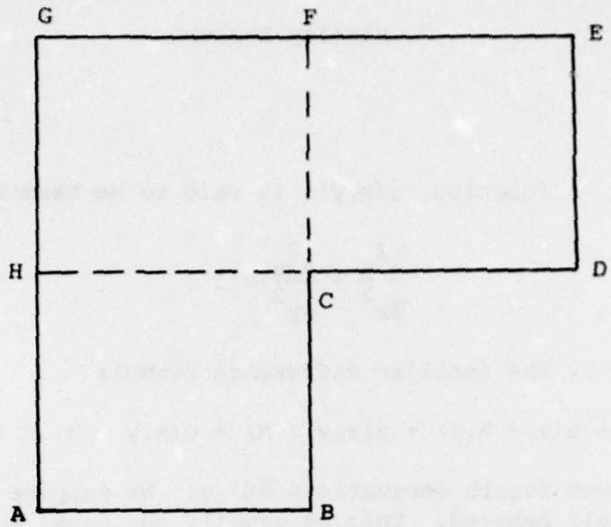

Figure 1

This was the solution of (1.2) inside the L-shaped region. As noted, the error of (1.2) as an approximation to (1.1) depends on fourth derivatives of u. In [2] and [3], which appeared six to four years before [1], Wasow and Lehman had shown that in the neighborhood of a reentrant corner (such as C in Fig. 1) one should expect to have an unbounded first derivative. With an unbounded first derivative, one cannot expect good behavior from fourth derivatives. So, in the neighborhood of C in Fig. 1, one should expect the solution of (1.2) (which was obtained in [1]) to be a poor approximation to the solution of (1.1).

<u>2. *Two lemmas*.</u> To get some comprehension of the results of Wasow and Lehman in [2] and [3], we use two lemmas, which we state here without proof.

<u>Lemma 2.1</u>. Let $a < b$. Let $f(a) = f(b) = 0$. Let $f(x)$ have almost everywhere a second derivative for $a \leq x \leq b$ which is of bounded variation. Then the Fourier series for $f(x)$ in the interval $a \leq x \leq b$,

$$f(x) = \sum_{m=1}^{\infty} D_m \sin \frac{\pi m(x-a)}{b-a} \tag{2.1}$$

with

$$D_m = \frac{2}{b-a} \int_a^b f(x) \sin \frac{\pi m(x-a)}{b-a}\, dx\,, \tag{2.2}$$

converges very rapidly, and a large number of the D_m can be calculated very quickly by means of the Fast Fourier Transform.

By "converges very rapidly" is meant that $|D_m|$ goes to zero at least of the order of m^{-3}. Thus one can truncate the series on the right of (2.1) after 500 terms and reasonably expect to get from six to eight significant decimal places correct. And the Fast Fourier Transform will enable one to calculate the needed 500 coefficients very quickly. The reasoning to establish this lemma is given in pp. 6-8 of [4].

Lemma 2.2. Let $u(x,y)$ be harmonic in a region, part of the boundary of which is a straight line segment. Let $u(x,y) = f(s)$ on this straight line segment, where s is length along the segment. Let $f(s)$ and its first n derivatives be continuous, and let the $(n+1)$-st derivative be bounded and continuous except at a set of points of measure zero. Then each partial derivative of $u(x,y)$ of order $\leq n$ has a continuous extension to the straight line boundary.

Thm. 2.3 on p. 27 of [5] states this for a special case. The truth of the lemma in general follows easily from the special case.

In the present report, we shall confine our attention to the case where the prescribed values of u around the boundary are quite smooth; say that the third derivative is bounded and continuous except at a set of measure zero. It is planned to write a sequel to [5] explaining how to handle a variety of irregularities along the boundary. Certain sorts of irregularities that could occur along the boundary can be "removed" by the methods given on pp. 221-222 of [6]. So it does not seem unduly restrictive to confine our attention to harmonic functions $u(x,y)$ which are quite smooth around the boundary. In the present report we do so.

Let $u(x,y)$ be such a harmonic function in the L-shaped region of Fig. 2.

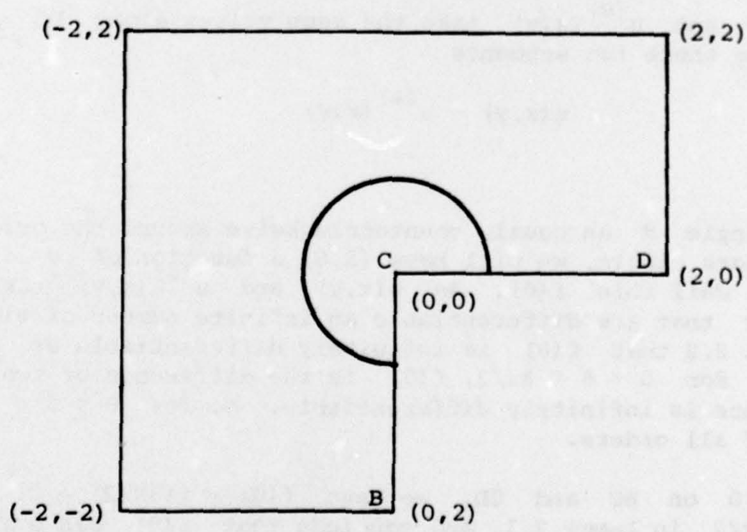

Figure 2

Indeed we will shortly specialize to prescribing that on the boundary in Fig. 2 we will have

$$u(x,y) = \frac{1}{2} \ln\{(x+1)^2 + (y-1)^2\} . \tag{2.3}$$

In Fig. 2 we have shown three-quarters of a circle of radius A and center at the origin. We undertake to determine the behavior of $u(x,y)$ inside the three-quarters circle.

Choose $u^{(s)}(x,y)$ a function which in the interior of the figure is harmonic in the neighborhood of the sides BC and CD, including the three-quarters circle (one can take A quite small if need be), and which takes the same values along BC and CD that are prescribed for $u(x,y)$. Instructions for finding such a function $u^{(s)}(x,y)$ are set forth in [5]. If we have prescribed the particular boundary conditions (2.3) for $u(x,y)$, such a function is

$$\frac{1}{4} \mathcal{R}\{3 \ln(z+1-i) + \ln(z+1+i) + \ln(z-1-i) - \ln(z-1+i)\} , \tag{2.4}$$

where we have taken

$$z = x + iy . \tag{2.5}$$

It can easily be verified that the right side of (2.4) satisfies (2.3) along the entire x-axis and the entire y-axis. Also (2.4) is harmonic except at the four points $z = \pm 1 \pm i$. To keep the three-quarters circle inside the region where (2.4) is harmonic, it suffices to take $A < \sqrt{2}$. We choose such a value for A, and proceed.

Since $u(x,y)$ and $u^{(s)}(x,y)$ take the same values along BC and CD, we conclude that along these two segments

$$u(x,y) - u^{(s)}(x,y) \tag{2.6}$$

is zero.

Measure the angle θ as usual, counterclockwise around the origin from CD. On the three-quarters circle, we will have (2.6) a function of θ only, since we have fixed A. Call this $f(\theta)$. As $u(x,y)$ and $u^{(s)}(x,y)$ take values along BC and CD that are differentiable an infinite number of times, it follows from Lemma 2.2 that $f(\theta)$ is infinitely differentiable as θ approaches $0+$ and $(3\pi/2)-$. For $0 < \theta < 3\pi/2$, $f(\theta)$ is the difference of two harmonic functions, and hence is infinitely differentiable. So for $0 \leq \theta \leq 3\pi/2$, $f(\theta)$ has derivatives of all orders.

As (2.6) is 0 on BC and CD, we have $f(0) = f(3\pi/2) = 0$. So we take $a = 0$ and $b = 3\pi/2$ in Lemma 2.1, and conclude that $f(\theta)$ has a rapidly converging Fourier series expansion for $0 \leq \theta \leq 3\pi/2$; put θ for x in (2.1). Because of the rapid convergence, we see that

$$(2.7) \qquad \sum_{m=1}^{\infty} D_m \left(\frac{r}{A}\right)^{2m/3} \sin \frac{2m\theta}{3}$$

is a harmonic function for $0 < r < A$ and $0 < \theta < 3\pi/2$. It equals $f(\theta)$ for $r = A$, and is 0 for $\theta = 0$ or $\theta = 3\pi/2$. But (2.6) satisfies these same conditions. As a harmonic function is uniquely determined in a region by its values around the boundary, we must have (2.6) equal to (2.7) inside and on the three-quarters circle. Solving for $u(x,y)$, we must have $u(x,y)$ equal to the sum of (2.4) and (2.7) inside and on the three-quarters circle.

For the particular boundary conditions which we have chosen (see (2.3)) we have $D_1 \neq 0$ in (2.7). So if we fix a value of θ, $0 < \theta < 3\pi/2$, and approach the origin along that ray, (2.7) will have an infinite first derivative. As (2.4) is harmonic inside the entire circle of radius A and center at the origin (we took $A < \sqrt{2}$), it has well behaved derivatives of all orders at C. So $u(x,y)$ must have an infinite derivative as r approaches zero.

Of course, if it had turned out that $D_1 = D_2 = D_4 = D_5 = 0$, then $u(x,y)$ would have had well behaved fourth derivatives. But it did not turn out that way. In [2] and [3], Wasow and Lehman made a study of the asymptotic behavior of harmonic functions near reentrant corners. Their studies were quite general, covering curved boundaries and a wide variety of conditions. The series they got were only asymptotic, but series like (2.7) were typical (except that (2.7) converges). Indeed, we are lucky with our particular problem, in that our $u^{(s)}(x,y)$ is harmonic in the neighborhood of the corner. More generally, $u^{(s)}(x,y)$ contributes additional complications, such as terms involving logarithms.

In view of this, one wonders why the authors of [1] managed to get such good results near the reentrant corner. This came about as follows. In order to be able to check if their procedure was giving the right answers, they took a problem in which the answers were known. They chose $u(x,y)$ a function that was well behaved over a much larger region than that shown in Fig. 1. From it, they read values around the boundary, and proceeded to solve, getting back $u(x,y)$ of course. Since they started with a function that was well behaved over a large region, including the reentrant corner, they insured that $D_1 = D_2 = D_4 = D_5 = 0$ in (2.7). So of course they had well behaved fourth derivatives, and (1.2) was an excellent approximation to (1.1), and their answers agreed closely with the true values. Very comforting for them, but very misleading for the reader. Had they used the boundary conditions (2.3), their answers would have been very poor near C. On the other hand, they probably did not have a way to get the correct answers for the boundary conditions (2.3), and so would not have known if they had good answers or not.

3. The solution inside a rectangle. This brings us to the crucial question. How does one get correct answers with boundary conditions like (2.3)? First we have to have a technique for carrying out a solution inside a rectangle, which we now explain. Given a rectangle with smooth boundary conditions prescribed around its perimeter, how does one determine a $u(x,y)$ which is harmonic in the interior and takes the prescribed values on the perimeter?

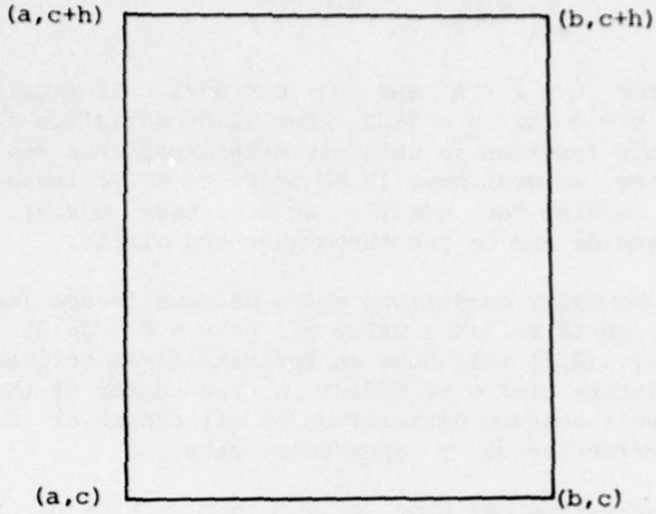

Figure 3

Consider

$$u^*(x,y) = u(x,y) - A - Bx - Cy - Dxy \; . \tag{3.1}$$

This is harmonic, and has smooth boundary conditions. It is easy to choose A, B, C, and D so that $u^*(x,y)$ takes the value zero at each corner of the rectangle. Along the top let $u^*(x,y) = f(x)$, for $a \le x \le b$. Our choice of A, B, C, and D assures that $f(a) = f(b) = 0$. Also, as we were assuming smooth boundary conditions, let us say that that assures that $f(x)$ has almost everywhere a second derivative for $a \le x \le b$ which is of bounded variation. So $f(x)$ satisfies the conditions of Lemma 2.1. We get its Fourier expansion, (2.1), with the D_m defined by (2.2). Consider

$$u_t(x,y) = \sum_{m=1}^{\infty} D_m \frac{\sinh \frac{\pi m(y-c)}{b-a}}{\sinh \frac{\pi m h}{b-a}} \sin \frac{\pi m(x-a)}{b-a} \; , \tag{3.2}$$

where h is the height of the rectangle and c is the value of y at the bottom of the rectangle. Clearly $u_t(x,y)$ is harmonic. It is zero along the left side of the rectangle $(x = a)$, it is zero along the right side of the rectangle $(x = b)$, it is zero along the bottom of the rectangle $(y = c)$, and it equals $f(x)$ along the top of the rectangle; that is, on the top it agrees with $u^*(x,y)$. We carry out an analogous construction for each of the other three sides of the rectangle, and add together the resulting four series. Since the sum agrees with $u^*(x,y)$ on the entire perimeter, it has to be equal to $u^*(x,y)$ throughout the rectangle. Then we determine $u(x,y)$ from (3.1).

Armed with this technique, let us return to the problem of Fig. 1. Values of $u(x,y)$ have been prescribed around the boundary (for example, see (2.3)). We guess values along CF. With the boundary conditions, this gives values

around the perimeter of the rectangle ABFG. As described just above, we get a harmonic function inside this rectangle which takes the prescribed boundary conditions. It gives us values along HC. With these and the boundary conditions, we have values around the perimeter of the rectangle HDEG. From these, we get values in the interior, including along the line CF. This will be an improvement over our first guess.

We repeat the process. In an actual calculation, with the conditions (2.3), it took about fifteen iterations for convergence. However, because the Fast Fourier Transform gets the D_m very quickly, the calculation to convergence did not take very long. However, it did not converge to the $u(x,y)$ we were seeking. Recall that in Lemma 2.1, it was required that $f(x)$ have a second derivative of bounded variation. But the $u(x,y)$ defined by conditions (2.3) has an infinite first derivative as one approaches C along CF.

This seems too bad. However, the procedure we just described is not without value. In fact, it will be the one we will use in the end, but with a slight modification. Our difficulty (refer to Fig. 1) is that, along the lines BF and HD, the function $u(x,y)$ that we are trying to determine does not have almost everywhere a second derivative of bounded variation. If we should try this procedure on a $u(x,y)$ which does have almost everywhere a second derivative of bounded variation along the lines BF and HD, we would succeed admirably in determining that $u(x,y)$, and in terms of rapidly converging Fourier series. All we need for $u(x,y)$ is to know its values around the boundary, and to be assured that it is sufficiently smooth along the lines BF and HD.

4. A slight modification. Recall that the $u(x,y)$ we are seeking to determine equals (2.4) plus (2.7) inside and on the three-quarters circle, and that (2.4) is very smooth along the lines BF and HD. Because of this, we will show that

$$u(x,y) - D_1\left(\frac{r}{A}\right)^{\frac{2}{3}} \sin\frac{2\theta}{3} - D_2\left(\frac{r}{A}\right)^{\frac{4}{3}} \sin\frac{4\theta}{3} \tag{4.1}$$

has a second derivative of bounded variation along both the lines BF and HD. Along BC and CD, (4.1) equals the right side of (2.3), which is very smooth. Inside and on the three-quarters circle, (4.1) equals (2.4) plus the remainder of the series (2.7), which is smooth enough. And from the three-quarters circle out to F or H, (4.1) is the sum of three harmonic functions out to a straight line border along which their boundary values are infinitely differentiable; by Lemma 2.2, all derivatives exist continuously out to the border.

If we could somehow determine the values of D_1 and D_2, we could determine (4.1) by the procedure of the previous section. We certainly can determine the values of (4.1) around the boundary; we had had to choose a value of A before the values of D_1 and D_2 could be defined (in fact, we had chosen $A = 1$ for our calculation), and the values of $u(x,y)$ are given by (2.3). Also, as we have just carefully ascertained, (4.1) has a second derivative of bounded variation along the lines BF and HD. Being given the values of D_1 and D_2, we could then calculate $u(x,y)$ from (4.1).

So we are faced with the problem of determining D_1 and D_2.

We remind the reader that a computer operates linearly. To calculate (4.1) by the procedure of the previous section, we would get the same numerical answers by either of the two following procedures.

(1) Apply the procedure to the total function (4.1).

(2) Apply the procedure first to $u(x,y)$, getting some Fourier expansions S^I, then apply the procedure to

$$\left(\frac{r}{A}\right)^{\frac{2}{3}} \sin \frac{2\theta}{3}, \tag{4.2}$$

getting some Fourier expansions S^{II}, then apply the procedure to

$$\left(\frac{r}{A}\right)^{\frac{4}{3}} \sin \frac{4\theta}{3}, \tag{4.3}$$

getting some Fourier expansions S^{III}, and finally combine the various Fourier expansions into

$$S^I - D_1 S^{II} - D_2 S^{III}. \tag{4.4}$$

Although S^I will be a poor representation of $u(x,y)$, as we observed in the previous section, and S^{II} and S^{III} will be poor representations of (4.2) and (4.3), for similar reasons, the combination (4.4) will be an excellent representation of (4.1), since the linearity of the computer assures that it comprises the same numbers that one would get by applying the procedure of the previous section to the entirety of (4.1).

With no more ado, let us proceed to determine S^I, S^{II}, and S^{III}. Considering D_1 and D_2 as two (as yet) unknown parameters, we can take (4.4) as representing (4.1). Subtracting (2.4) from (4.4), we will have a representation of

$$u(x,y) - u^{(s)}(x,y) - D_1\left(\frac{r}{A}\right)^{\frac{2}{3}} \sin \frac{2\theta}{3} - D_2\left(\frac{r}{A}\right)^{\frac{4}{3}} \sin \frac{4\theta}{3}. \tag{4.5}$$

That is, using (4.4) minus (2.4), we can actually calculate values of (4.5) at any points of the L-shaped region of Fig. 2, except that the values will come out as linear combinations of D_1 and D_2.

Observe that (4.5) is zero on both the lines BC and CD, since (2.6) was. So, by the same method that we used to get the expansion (2.7) for (2.6) inside and on the three-quarters circle, we can get an expansion like (2.7) for (4.5). Obviously, this expansion has to consist of (2.7) with the first two terms deleted. So, when we take $m = 1$ and 2 in (2.2) to get D_1 and D_2, we must

get the value zero. But, as the values of $f(x)$ in (2.2) are taken from (4.4) minus (2.4), the numerical quadratures to determine (2.2) must yield linear combinations of D_1 and D_2. Putting these linear combinations equal to zero for $m = 1$ and $m = 2$ gives us two simultaneous linear equations for D_1 and D_2. We solve these. Putting the solutions into (4.4) gives Fourier expansions for (4.1). But now we know D_1 and D_2, and so can calculate $u(x,y)$ from (4.1).

5. Acknowledgements. In a Part II, we will report numerical results for the problem considered above, and results for other problems that can be handled by similar techniques. I wish to express my gratitude to Gershon Kedem for his assistance with these activities. He carried out the needed programming, and supervised the actual calculations. He also suggested simplifications, and helped me get my thoughts in order and keep track of the details.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

It has been known for quite a while that if a function $u(x,y)$ is harmonic in a region with reentrant corners, there are almost certainly infinite discontinuities of the first derivative of u in the neighborhood of the reentrant corner (or corners). Simple examples are for an L-shaped region or T-shaped region. Some instances of these have been treated by conformally mapping the region into the interior of a rectangle. Attempts to solve the problem as first posed by a finite difference scheme or a finite element scheme

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20. ABSTRACT (Cont'd.)

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